'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester [Special] Examination, 2023

GE2-P2-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE4-II, MATHGE4-III, MATHGE4-V. The candidates are required to answer any *one* from the *three* courses. Candidates should mention it clearly on the Answer Book.

MATHGE4-II

ALGEBRA

GROUP-A

1. Answer any ***four*** questions: 3×4 = 12

(a) If n is a positive integer, show that $\left(1+\frac{1}{n+1}\right)^{n+1} > \left(1+\frac{1}{n}\right)^{n}$. 3

(b) Show that the sum of the squares of all values of $(\sqrt{3}+i)^{3/7}$ is 0. 3

(c) Find the remainder when $1!+2!+3!+\cdots\cdots+50!$ is divided by 15. 3

(d) Show that if d is an eigen value of a non-singular matrix A, then λ^{-1} is an eigen value of A^{-1}. 3

(e) If α, β, γ be the roots of $x^3+px^2+qx+r=0$, find the values of (i) $\sum \alpha^2\beta$, (ii) $\sum \frac{1}{\alpha\beta}$. 3

(f) Obtain a row echelon matrix which is row equivalent to $\begin{pmatrix} 1 & 2 & 1 & -3 \\ 2 & 4 & 3 & 1 \\ 3 & 6 & 4 & -1 \end{pmatrix}$. 3

GROUP-B

Answer any *four* questions 6×4 = 24

2. Show that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan 4\theta}$. 6

3. Solve the equation $x^4-4x^3-4x^2-4x-5=0$, whose two roots α, β satisfies $2\alpha+\beta=3$. 6

4. Find the real value of λ such that the following system has a non-zero solution: 6

$$x+2y+3z=\lambda x$$
$$3x+y+2z=\lambda y$$
$$2x+3y+z=\lambda z$$

5. Find all eigen values and corresponding eigen vectors of the matrix 6

$$A=\begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$$

6. Use division algorithm to prove that the square of odd integers is of the form $8k+1$, where k is an integer. 6

7. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$, and hence find A^{-1}. 6

GROUP-C

Answer any *two* questions $12\times 2=24$

8. (a) If $d=\gcd(a,\, m)$, then show that $ax\equiv ay \pmod m$ iff $x\equiv y \left(\bmod \frac{m}{d}\right)$. 6

(b) Solve by Ferrari's method $x^4-6x^2+16x-15=0$. 6

9. (a) Prove that the product of any *m* consecutive integers is divisible by *m*. 6

(b) If *R* and *S* be equivalence relations on a set *A*, prove that R^{-1} is an equivalence relation on *A* and $R\cap S$ is also an equivalence relation on *A*. 6

10.(a) Obtain the fully reduced normal form of the matrix $\begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 4 & 6 \\ 3 & 0 & 7 & 9 \\ 2 & 6 & 1 & 1 \end{pmatrix}$. 4

(b) Apply Descarte's rule of signs to find the nature of roots of $x^7+x^5-x^3=0$. 4

(c) Find two integers *U* and *V* satisfying $63U+55V=1$. 4

11.(a) If the roots of $x^3+2x^2+3x+1=0$ are α, β, γ, then find the equation whose roots are $\left(\frac{1}{\beta^2}+\frac{1}{\gamma^2}-\frac{1}{\alpha^2}\right)$, $\left(\frac{1}{\gamma^2}+\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right)$ and $\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}-\frac{1}{\gamma^2}\right)$. 6

(b) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then prove that 6

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{3}{2}$$

MATHGE4-III

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4=12$

(a) Find $\dfrac{1}{(D+1)(D+2)^2}\{e^{-2x}+e^x.x+3e^{-x}\}$.

(b) Locate and classify the singular points of equation

$$x^2(x^2-4)\frac{d^2y}{dx^2}+3x^3\frac{dy}{dx}+4y=0$$

(c) Show that e^{3x}, xe^{3x} and x^2e^{3x} are the solutions of $\dfrac{d^3y}{dx^3}-9\dfrac{d^2y}{dx^2}+27\dfrac{dy}{dx}-27y=0$. Are they linearly independent? Justify.

(d) Find the volume of the parallelopiped whose three concurrent sides are represented by the vectors: $\hat{i}+2\hat{j}+3\hat{k}$, $3\hat{i}+7\hat{j}-4\hat{k}$, $\hat{i}-5\hat{j}+3\hat{k}$.

(e) If ϕ is a harmonic function, show that gradϕ is a solenoidal vector field.

(f) If $\vec{a}, \vec{b}, \vec{c}$ be non-coplanar, prove that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also non-coplanar. 3

GROUP-B

Answer any *four* questions from the following $6\times4=24$

2. Solve: 6

$$\frac{dx}{dt}+4x+3y=t$$

$$\frac{dy}{dt}+2x+5y=e^t$$

3. Solve: $\dfrac{d^3y}{dx^3}-5\dfrac{d^2y}{dx^2}+8\dfrac{dy}{dx}-4y=e^{2x}+e^x+3e^{-x}$ 6

4. Solve by method of variation of parameters $\dfrac{d^3y}{dx^3}+\dfrac{dy}{dx}=\operatorname{cosec} x$. 6

5. Solve the differential equation by reducing to normal form: 6

$$\frac{d^2y}{dx^2}-\frac{2}{x}\frac{dy}{dx}+\left(a^2+\frac{2}{x^2}\right)y=0$$

6. Show that the vector field $\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ is irrotational. Find the scalar function $\phi(x, y, z)$ such that $\vec{\nabla}\phi=\vec{F}$. 6

7. Evaluate $\iint_S \vec{F}\cdot\hat{n}\; dS$ 6

where $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ and S is the surface of the cube bounded by $x=0$, $x=1$; $y=0$, $y=1$; $z=0$, $z=1$.

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Solve: $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2e^{3x}$ 6

(b) Solve: $(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=8(5+2x)^2$ 6

9. (a) Solve by the method of undetermined coefficient $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+3y=x^3+\sin x$. 6

(b) Find the general solution of $(1+x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=(1+x)^2$ by the method of variation of parameters, it is given that $y=x$ and $y=e^{-x}$ are two linearly independent solutions of the corresponding homogeneous equation. 6

10.(a) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=xy\hat{i}+(x^2+y^2)\hat{j}$ and C is the x-axis from $x=2$ to $x=4$ and the straight line from $y=0$ to $y=12$. 6

(b) Find the work done in moving a particle in a force field $\vec{F}=3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$ along the line joining (0, 0, 0) to (2, 1, 3). 6

11.(a) A particle moves along a curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$, where t is the time. Find the component of its velocity and acceleration at $t=1$ in direction $\vec{a}=\hat{i}-3\hat{j}+2\hat{k}$. 5

(b) Solve: 5

$$\frac{d^2x}{dt^2}+4x+y=te^{3t}$$

$$\frac{d^2y}{dt^2}+y-2x=\cos^2 t$$

(c) Find $\frac{d\vec{A}}{dt}\times\frac{d^2\vec{A}}{dt^2}$, where $\vec{A}=t^3\hat{i}-(t^2-2)\hat{j}+(t^3-t)\hat{k}$. 2

MATHGE4-V

Numerical Methods

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) If $\sqrt{3}$ be represented by 1.732, find relative error and percentage error.

(b) What is pivoting? State the sufficient condition of convergence of Gauss-Seidel iteration.

(c) What are the merits and demerits of Lagrange's interpolation formula?

(d) Write down the order of convergence of
   (i) Regula-Falsi method
   (ii) Newton-Raphson method
   (iii) Secant method.

(e) Show that $\Delta \log f(x) = \log\left\{1 + \dfrac{\Delta f(x)}{f(x)}\right\}$.

(f) What is the geometrical meaning of Newton-Raphson method?

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Using Lagrange's interpolation formula, find $f(6.60)$ from the following data:

x	6.54	6.58	6.59	6.61	6.64
$f(x)$	2.8156	2.8182	2.8189	2.8202	2.8222

3. Compute the values of the unknowns in the system of equation by Gauss-Jordan's matrix inversion method:

$$2x_1 - 3x_2 + 4x_3 = 8$$
$$x_1 + x_2 + 4x_3 = 15$$
$$3x_1 + 4x_2 - x_3 = 8$$

4. Evaluate $\int_0^{\pi/2} \sqrt{\cos x}\, dx$ by Weddle's rule, correct upto three significant figures, taking six intervals.

5. Establish the Picard's iteration formula for $\dfrac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$.

6. Solve by Gauss-elimination method:

$$x + 3y + 2z = 5$$
$$2x - y + z = -1$$
$$x + 2y + 3z = 2$$

7. For any positive k, show that $\nabla^k y_n = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{n-i}$, ∇ being the backward difference operator.

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Find the number of multiplications and divison's for solving a system of n linear equations having n unknowns using Gauss-elimination method. 6

(b) What are partial and complete pivoting in Gauss-elimination method? 3

(c) Find the missing term in the following table: 3

x	16	18	20	22	24	26
$f(x)$	39	85	-	151	233	388

9. (a) If $f(x)$ is a polynomial of degree 2, prove that 6

$$\int_0^1 f(x)\,dx = \frac{1}{12}[5f(0) + 8f(1) - f(2)]$$

(b) Explain modified Euler's method, for solving first order differential equation of the form 6

$$\frac{dy}{dx} = f(x, y) \quad , \quad y(x_0) = y_0$$

10.(a) Compute the root of the equation $2x - 3\sin x - 5 = 0$ by Regula-Falsi Method correct upto three decimal places. 6

(b) Define the operator Δ, prove that $\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n n!}{x(x+1)(x+2)\cdots\cdots(x+n)}$. 1+5

11.(a) Evaluate $\int_{0.1}^{0.7} (e^x + 2x)\,dx$, by Simpson's $1/3^{\text{rd}}$ rule, taking $h = 0.1$ and correct upto five decimal places. 6

(b) Evaluate $f(1.1)$ from the table below: 6

x	0	1	2	3	4	5
$f(x)$	0	3	8	15	24	35

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B.Sc. Honours 4th Semester Examination, 2023

GE2-P2-MATHEMATICS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE4-I, MATHGE4-II, MATHGE4-III, MATHGE4-IV & MATHGE4-V. The candidates are required to answer any *one* from the *five* courses. Candidates should mention it clearly on the Answer Book.

MATHGE4-I

CALCULUS, GEONETRY AND DIFFERENTIAL EQUATION

GROUP-A

1. Answer any ***four*** questions from the following: $3\times 4=12$

(a) Find the length of the curve, $x=t^2$, $y=\sin t-t\cos t$, $z=\cos t+t\sin t$ from $t=0$ to $t=\pi$. 3

(b) Find the asymptotes of the curve, 3

$$x=\frac{t^2+1}{t^2-1} \quad , \quad y=\frac{t^2}{t-1}$$

(c) Evaluate: $\lim\limits_{x\to 0}\dfrac{x-\sin^{-1}x}{\sin^3 x}$ 3

(d) Show that the distance of a point (*x*, *y*) from the origin is an invariant under rotation of axes. 3

(e) Transform the equation $x^3=y^2(2a-x)$ into the polar equation. 3

(f) Find the order and degree of the differential equation: $\left(\dfrac{d^3y}{dx^3}\right)^2=\sqrt{y+\left(\dfrac{dy}{dx}\right)^3}$ 3

GROUP-B

Answer any *four* questions from the following $6\times 4=24$

2. If $\ln y=\tan^{-1}x$, then show that 6

$$(1+x^2)y_{n+2}+(2nx+2x-1)\,y_{n+1}+n(n+1)\,y_n=0$$

3. Show that the curve $y(x^2+a^2)=a^2x$ has three points of inflexion lie on the line $x=4y$. 6

4. If $I_n = \int_0^1 x^n \tan^{-1} x\, dx,\ n > 2$, prove that $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$. 6

5. Find the volume of the solid of revolution obtained by revolving $x^{2|3} + y^{2|3} = a^{2|3}$ about x-axis. 6

6. Solve the differential equation: $x\frac{dy}{dx} + y = y^2 \log x$ 6

7. Reduce the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ to its canonical form and show that it represents a hyperbola. 6

GROUP-C

Answer any *two* questions from the following $12\times2 = 24$

8. (a) Solve: $(x^2 y - 2xy^2)\,dx - (x^3 - 3x^2 y)\,dy = 0$. 4

(b) Obtain the differential equation corresponding to $(x-a)^2 + (y-b)^2 = 9$, where a, b are arbitrary parameters. 4

(c) Solve: $x^2 dy + (xy + y^2)dx = 0$ 4

9. (a) Show that the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$, if $(l\cos\alpha - ep)^2 + l^2 \sin^2\alpha = p^2$. 6

(b) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and which touches the plane $2x + 2y - z = 15$. 6

10.(a) Find the equation of the cylinder, whose generators are parallel to the straight line $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 9$, $z = 1$. 6

(b) Reduce the equation $x^2 + y^2 + z^2 - 2xy - 2yz + 2zx + x - 4y + z + 1 = 0$ to its canonical form and determine the nature of the quadric. 6

11.(a) A circle moves with its centre on the parabola $y^2 = 4ax$ and always passes through the vertex of the parabola. Show that the envelope of the circle is the curve $x^3 + y^2(x + 2a) = 0$. 6

(b) Find the asymptotes of the curve $2x^3 + 3x^2 y - 3xy^2 - 2y^3 + 3x^2 - 3y^2 + y - 3 = 0$. 6

MATHGE4-II

ALGEBRA

GROUP-A

1. Answer any ***four*** questions: $3\times4 = 12$

(a) If n is a positive integer, show that $\left(1 + \frac{1}{n+1}\right)^{n+1} > \left(1 + \frac{1}{n}\right)^n$. 3

(b) Show that the sum of the squares of all values of $(\sqrt{3} + i)^{3/7}$ is 0. 3

(c) Find the remainder when $1!+2!+3!+\cdots\cdots+50!$ is divided by 15. 3

(d) Show that if *d* is an eigen value of a non-singular matrix *A*, then λ^{-1} is an eigen value of A^{-1}. 3

(e) If α, β, γ be the roots of $x^3+px^2+qx+r=0$, find the values of (i) $\sum \alpha^2\beta$, (ii) $\sum \frac{1}{\alpha\beta}$. 3

(f) Obtain a row echelon matrix which is row equivalent to $\begin{pmatrix} 1 & 2 & 1 & -3 \\ 2 & 4 & 3 & 1 \\ 3 & 6 & 4 & -1 \end{pmatrix}$. 3

GROUP-B

Answer any *four* questions $6\times4=24$

2. Show that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan 4\theta}$. 6

3. Solve the equation $x^4-4x^3-4x^2-4x-5=0$, whose two roots α, β satisfies $2\alpha+\beta=3$. 6

4. Find the real value of λ such that the following system has a non-zero solution: 6

$$x+2y+3z=\lambda x$$
$$3x+y+2z=\lambda y$$
$$2x+3y+z=\lambda z$$

5. Find all eigen values and corresponding eigen vectors of the matrix 6

$$A=\begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$$

6. Use division algorithm to prove that the square of odd integers is of the form $8k+1$, where *k* is an integer. 6

7. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$, and hence find A^{-1}. 6

GROUP-C

Answer any *two* questions $12\times2=24$

8. (a) If $d=\gcd(a, m)$, then show that $ax \equiv ay \pmod{m}$ iff $x \equiv y \left(\bmod \frac{m}{d}\right)$. 6

(b) Solve by Ferrari's method $x^4-6x^2+16x-15=0$. 6

9. (a) Prove that the product of any *m* consecutive integers is divisible by *m*. 6

(b) If *R* and *S* be equivalence relations on a set *A*, prove that R^{-1} is an equivalence relation on *A* and $R \cap S$ is also an equivalence relation on *A*. 6

10.(a) Obtain the fully reduced normal form of the matrix $\begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 4 & 6 \\ 3 & 0 & 7 & 9 \\ 2 & 6 & 1 & 1 \end{pmatrix}$. 4

(b) Apply Descarte's rule of signs to find the nature of roots of $x^7 + x^5 - x^3 = 0$. 4

(c) Find two integers U and V satisfying $63U + 55V = 1$. 4

11.(a) If the roots of $x^3 + 2x^2 + 3x + 1 = 0$ are α, β, γ, then find the equation whose roots are $\left(\frac{1}{\beta^2} + \frac{1}{\gamma^2} - \frac{1}{\alpha^2}\right)$, $\left(\frac{1}{\gamma^2} + \frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)$ and $\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}\right)$. 6

(b) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that 6

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{3}{2}$$

MATHGE4-III

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Find $\frac{1}{(D+1)(D+2)^2}\{e^{-2x} + e^x.x + 3e^{-x}\}$.

(b) Locate and classify the singular points of equation

$$x^2(x^2-4)\frac{d^2y}{dx^2} + 3x^3\frac{dy}{dx} + 4y = 0$$

(c) Show that e^{3x}, xe^{3x} and x^2e^{3x} are the solutions of $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 27\frac{dy}{dx} - 27y = 0$. Are they linearly independent? Justify.

(d) Find the volume of the parallelopiped whose three concurrent sides are represented by the vectors: $\hat{i} + 2\hat{j} + 3\hat{k}$, $3\hat{i} + 7\hat{j} - 4\hat{k}$, $\hat{i} - 5\hat{j} + 3\hat{k}$.

(e) If ϕ is a harmonic function, show that gradϕ is a solenoidal vector field.

(f) If $\vec{a}, \vec{b}, \vec{c}$ be non-coplanar, prove that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also non-coplanar. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Solve: 6

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

3. Solve: $\frac{d^3y}{dx^3}-5\frac{d^2y}{dx^2}+8\frac{dy}{dx}-4y=e^{2x}+e^x+3e^{-x}$ 6

4. Solve by method of variation of parameters $\frac{d^3y}{dx^3}+\frac{dy}{dx}=\text{cosec}\, x$. 6

5. Solve the differential equation by reducing to normal form: 6

$$\frac{d^2y}{dx^2}-\frac{2}{x}\frac{dy}{dx}+\left(a^2+\frac{2}{x^2}\right)y=0$$

6. Show that the vector field $\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ is irrotational. Find the scalar function $\phi(x, y, z)$ such that $\vec{\nabla}\phi=\vec{F}$. 6

7. Evaluate $\iint\limits_S \vec{F}\cdot\hat{n}\, dS$ 6

where $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ and S is the surface of the cube bounded by $x=0,\ x=1;\ y=0,\ y=1;\ z=0,\ z=1$.

GROUP-C

Answer any *two* questions from the following $12\times2=24$

8. (a) Solve: $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2e^{3x}$ 6

(b) Solve: $(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=8(5+2x)^2$ 6

9. (a) Solve by the method of undetermined coefficient $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+3y=x^3+\sin x$. 6

(b) Find the general solution of $(1+x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=(1+x)^2$ by the method of variation of parameters, it is given that $y=x$ and $y=e^{-x}$ are two linearly independent solutions of the corresponding homogeneous equation. 6

10.(a) Evaluate $\int\limits_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=xy\hat{i}+(x^2+y^2)\hat{j}$ and C is the x-axis from $x=2$ to $x=4$ and the straight line from $y=0$ to $y=12$. 6

(b) Find the work done in moving a particle in a force field $\vec{F}=3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$ along the line joining (0, 0, 0) to (2, 1, 3). 6

11.(a) A particle moves along a curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$, where t is the time. Find the component of its velocity and acceleration at $t=1$ in direction $\vec{a}=\hat{i}-3\hat{j}+2\hat{k}$. 5

(b) Solve: $\frac{d^2x}{dt^2}+4x+y=te^{3t}$ 5

$\frac{d^2y}{dt^2}+y-2x=\cos^2 t$

(c) Find $\frac{d\vec{A}}{dt}\times\frac{d^2\vec{A}}{dt^2}$, where $\vec{A}=t^3\hat{i}-(t^2-2)\hat{j}+(t^3-t)\hat{k}$. 2

MATHGE4-IV

GROUP THEORY

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Prove that in a group $G, (ab)^{-1}=b^{-1}a^{-1}$ for all $a, b\in G$. 3

2. Show that the set $S=\{e, \rho_1, \rho_2, \rho_3\}$ of permutations forms an abelian group with respect to 'multiplication of permutations', where 3

$e=\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$, $\rho_1=\begin{pmatrix}1&2&3&4\\2&1&3&4\end{pmatrix}$, $\rho_2=\begin{pmatrix}1&2&3&4\\1&2&4&3\end{pmatrix}$, $\rho_3=\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$

3. Find all elements of order 10 in the group $(\mathbb{Z}_{30}, +_{30})$. 3

4. Let G be a group and H, K be subgroups of G. Then show that $H\cap K$ is a subgroup of G. 3

5. Find all cyclic subgroups of the group $V=\{e, a, b, c\}$, where V is the Klein's 4-group. 3

6. Let $\mathbb{R}^+$ be the group of positive real numbers under multiplication and $\mathbb{R}$ the group of all real numbers under addition. Then show that the map $\theta:\mathbb{R}^+\longrightarrow\mathbb{R}$ such that $\theta(x)=\log_e x$ is an isomorphism. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Let G be a group and H be a subgroup of G. Prove that any two left cosets of H in G are either identical or they have no common element. 6

8. Prove that every group of prime order is cyclic. 6

9. Let H be a subgroup of a group G. Prove that the relation ρ defined on G by "$a\rho b$ if and only if $a^{-1}b\in H$" for $a, b\in G$ is an equivalence relation on G. 6

10. Prove that a subgroup H of a group G is normal in G if and only if $g^{-1}hg\in H$ for all $h\in H$, $g\in G$. 6

11. Prove that a group homomorphism $f: G\to G'$ is one-one if and only if $\ker f=\{e\}$. 6

12. Let $f: G\to G'$ be a group homomorphism. Let $a\in G$ be such that $O(a)=n$ and $O(f(a))=m$. Prove that $O(f(a))\,|\,O(a)$ and f is one-one iff $m=n$. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Prove that a finite group of order n is cyclic if and only if there exists an element b in G such that $O(b)=n$. 6

(b) Let $G=\{1, \omega, \omega^2, -1, -\omega, -\omega^2\}$ where $\omega=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. Prove that G is a cyclic group under multiplication. 6

14.(a) Define centre of a group G. Prove that the centre $Z(G)$ of a group G is a normal subgroup of G. 6

(b) Let G and G' be two groups and $\phi: G\to G'$ be a homomorphism. Prove that $\phi(G)$ is a subgroup of G'. 6

15.(a) Let H and K be subgroups of a group G. Then show that HK is a subgroup of G if and only if $HK = KH$. 6

(b) Prove that a finite semigroup in which cancellation laws hold is a group. 6

16.(a) If H and K are finite subgroups of a group G, then show that 7

$$O(HK)=\frac{O(H)\,O(K)}{O(H\cap K)}$$

(b) Find all subgroups of the group $\mathbb{Z}$ of integers. 5

MATHGE4-V

NUMERICAL METHODS

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) If $\sqrt{3}$ be represented by 1.732, find relative error and percentage error.

(b) What is pivoting? State the sufficient condition of convergence of Gauss-Seidel iteration.

(c) What are the merits and demerits of Lagrange's interpolation formula?

(d) Write down the order of convergence of

(i) Regula-Falsi method

(ii) Newton-Raphson method

(iii) Secant method.

(e) Show that $\Delta\log f(x)=\log\left\{1+\frac{\Delta f(x)}{f(x)}\right\}$.

(f) What is the geometrical meaning of Newton-Raphson method?

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Using Lagrange's interpolation formula, find $f(6.60)$ from the following data:

x	6.54	6.58	6.59	6.61	6.64
$f(x)$	2.8156	2.8182	2.8189	2.8202	2.8222

3. Compute the values of the unknowns in the system of equation by Gauss-Jordan's matrix inversion method:

$$2x_1 - 3x_2 + 4x_3 = 8$$
$$x_1 + x_2 + 4x_3 = 15$$
$$3x_1 + 4x_2 - x_3 = 8$$

4. Evaluate $\int_0^{\pi/2} \sqrt{\cos x}\, dx$ by Weddle's rule, correct upto three significant figures, taking six intervals.

5. Establish the Picard's iteration formula for $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$.

6. Solve by Gauss-elimination method:

$$x + 3y + 2z = 5$$
$$2x - y + z = -1$$
$$x + 2y + 3z = 2$$

7. For any positive k, show that $\nabla^k y_n = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{n-i}$, ∇ being the backward difference operator.

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Find the number of multiplications and divison's for solving a system of n linear equations having n unknowns using Gauss-elimination method. 6

(b) What are partial and complete pivoting in Gauss-elimination method? 3

(c) Find the missing term in the following table: 3

x	16	18	20	22	24	26
$f(x)$	39	85	-	151	233	388

9. (a) If $f(x)$ is a polynomial of degree 2, prove that 6

$$\int_0^1 f(x)\, dx = \frac{1}{12}[5f(0) + 8f(1) - f(2)]$$

(b) Explain modified Euler's method, for solving first order differential equation of the form 6

$$\frac{dy}{dx} = f(x, y) \quad , \quad y(x_0) = y_0$$

10.(a) Compute the root of the equation $2x - 3\sin x - 5 = 0$ by Regula-Falsi Method correct upto three decimal places. 6

(b) Define the operator Δ, prove that $\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n n!}{x(x+1)(x+2)\cdots\cdots(x+n)}$. 1+5

11.(a) Evaluate $\int_{0.1}^{0.7}(e^x + 2x)\,dx$, by Simpson's 1/3$^{\text{rd}}$ rule, taking $h = 0.1$ and correct upto five decimal places. 6

(b) Evaluate $f(1.1)$ from the table below: 6

x	0	1	2	3	4	5
$f(x)$	0	3	8	15	24	35

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